M.Sc. Mathematics 1 Semester Paper I: Real Analysis I

Course Code: Math: 101

Unit-l

- Point wise and uniform convergence, Cauchy criterion for uniform convergence.
- Weierstrass M-test. Abel's and Dirichlet's test for uniform convergence.
- Uniform convergence and continuity. uniform convergence and Riemann Stieltjes Integration, uniform convergence and differentiation.

Unit-II

- Weierstrass approximation theorem. Power series, Uniqueness theorem for Power series, Abel's and Taylor's theorems.
- Definition and existence of Riemann -- Stieltjes integral.
- Properties of the integral, integration and differentiation.

Unit-111

- The fundamental theorem of calculus.
- Integration of vector valued functions.
- Rectifiable curves.

Unit-IV

- Linear transformations, differentiation. partial derivatives, continuity of partial derivatives.
- Change in the order of partial derivative, the implicit functions, inverse function. jacobians.
- Maxima and Minima, Lagrange's multipliers, Rank Theorem, Determinants.

Paper II: Advanced Algebra

Course Code: Math-102

Unit-1

- The Sylow theorems, application of Sylow theory, Direct products
- Classification of finite abelian groups, the Jordan-Holder theorem
- Solvable group Example of Solvable group.

Unit-II

- Definition and examples of Rings, some special classes of Rings.
- Fields, ideals and quotient rings, ring homomorphism.
- Prime and maximal ideals, field of quotients.

Unit-III

- Euclidean rings.
- Definition and examples.
- Orthonormal sets and bases.

Unit-IV

- Annihilators, Projections.
- Hilbert Space.
- Linear functional on Hilbert spaces.

Suggested Books

I.I.N. Herstein, Topics in Algebra (Second Edition), John Wiley & Sons, New Yo 2.P.B. Bhattacharya, S.k. Jain, S.R. Nagpaul, Basic Abstract Algebra (Second Editi Cambridge University Press, 1994.

Paper III: Ordinary Differential Equations

Course Code: Math-103

Unit-I

- . Some concept from real function theory
- The Fundamental existence and uniqueness theorem
- Dependence of solutions on initial conditions and on the function
- Basic theory of the homogeneous linear system, further theory of the homogeneous linear system.

Unit-II

- The non-homogeneous linear system.
- Basic theory of the nth-order homogeneous linear differential equation, the nth-order non-homogeneous linear equation.
- Sturm-liouville problems
- Orthogonality of characteristic functions

Unit-III

- The expansion of a function in a series of orthonormal functions.,
- The separation theorem, Sturm's fundamental theorem.
- Conditions for Oscillatory or non Oscillatory solutions.
- First and second comparison theorems.
- Sturm oscillation theorems, application to Sturm-Liouville system.

Unit-IV

- Phase plane, paths and critical points.
- Critical points and paths of linear system.
- Critical point and path of non linear system.
- Limit cycles and periodic solutions.

- 1. S.L. Ross, Differential Equations. Third Edition, John Wiley & Sons.
- 2. M.D. Raisinghania, Advanced Differential Equation, S. Chand Publication.

Paper IV: Operation Research-I

Course Code: Math-104

Unit 1

- LPP. Formulations and graphical solutions, Feasible, basic feasible and optimal solutions
- Convex concave sets,
- Simplex method, Big-M method, Two-phase method, Degeneracy unrestricted variables. unbounded solutions.
- Duality in LPP, Dual Problem, Primal Dual relationships, Dual Simplex method.

Unit-11

- Mathematical formulation, basic feasible solution of transportation problem.
- North-West corner method
- Least cost and Revised Simplex Method.
- Vogel's approximation method, unbalanced TP.
- Optimality test of BFS, UV-method, matrix method, Row minima and column minima methods. Degeneracy in TP.

Unit-III

- The assignment model, Hungarian method, travelling sales man problem.
- Revised Simplex Method.
- Formulation of GP, Graphical goal attainment method. Simplex method for GPP.

Unit-IV

- Two person, Zero --sum games, the maxmin and minimax principle.
- Pure strategy, mixed strategies, graphical solution of 2Xn and mX2 games.
- Dominance property, general solution of mxn rectangular games.
- Linear programming problem of GP.

- S.D. Sharma, Operations research, Kedar Nath Ram Nath & Co.14 Edition 2004.
- 2. Kanti Swarup, P.K. Gupta and Manmohan, Operations research, Sultan chand & Se Edition.

Paper V: Fluid Dynamics

Course Code: AUMath-105

Unit-I

- Continuum Hypothesis, Newton's Law of viscosity.
- Some Cartesian, tensor notations, general analysis of fluid motion.
- Thermal conductivity, generalized heat conduction.
- Equation of state, equation of continuity. Navier-Stokes equations (equation of motion, equation of energy).

Unit-II

- Stream lines and path lines, Vorticity and circulation.
- Dynamical similarity (Reynold's Law).
- Steady motion between parallel plates (a) Velocity distribution (b) Temperature distribution.

Unit-III

- Plane couette flow, plane poiseuille flow. generalized plane couette flow.
- Flow in a circular pipe (hagen- poiseuille flow) (a) Velocity distribution (b) Temperature distribution.
- Flow between two concentric rotating cylinder (couette flow)(a) Velocity distribution
 - (b) Temperature distribution.

Unit-IV

- Flow due to a plane wall suddenly set in motion, flow due to an oscillating plane wall, Plane couette flow with transpiration cooling.
- Theory of lubrication, characteristics boundary layer parameters, steady flow past a fixed sphere: Stokes equation and Oseen's equation of flow.

Suggested Books:

1. J.L. Bansal, Viscous Fluid Dynamics, Oxford and IBH publishing Co. Pvt. Ltd. (1977). 2.F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers & Distributors (2000).

M.Sc. Mathematics 2 Semester

Paper I: Real Analysis II

Course Code: Math-201

Unit-1

- Introduction. Outer Measure, Measurable sets and Lebesgue measure,
- A non measurable set, Measurable functions.
- Littlewood's three principles.

Unit-II

- The Riemann integral
- The Lebesgue integral of a bounded function over a set of finite measure.
- The integral of nonnegative function
- The general Lebesgue integral.
- Convergence in measure.

Unit-III

- Differentiation of monotone functions.
- Functions of bounded variation.
- Differentiation of an integral
- Absolute continuity
- Convex functions.

Unit-IV

- The L' spaces.
- The Minkowski and Holder inequalities.
- Convergence and completeness.
- Approximation in L'. bounded linear functions on the LP spaces.

Suggested Books

1. H.L. Royden, Real Analysis, Third Edition. Prentice Hall if India, Private Limited, Ne Delhi, 1995.

Paper II: Advanced Algebra-II

Course Code: AUMath-202

Unit-I

- Definition and examples.
- Sub modules and direct sums
- Homeomorphism and quotient modules.
- Completely reducible module, free module

Unit-11

- Irreducible polynomials and Eisenstein criterion.
- Algebraic extensions, algebraically closed field, splitting fields
- Normal extensions, multiple roots, finite fields separable extensions.

Unit-III

- Automorphism groups and fixed fields.
- Fundamental theorem of Galois theory, fundamental theorem of algebra.

Unit-IV

- Roots of unity and cyclotomic polynomials
- Cyclic extensions
- Polynomial solvable by radicals
- Symmetric functions

- 1. P.B. Bhattacharya, S.k. Jain, S.R. Nagpaul, Basic Abstract Algebra (Second Edition),
 - Cambridge University Press, 1994.
- 2. Joseph A. Gallian. Contemporary Abstract Algebra (Eighth Edition

Paper III: Partial Differential Equations

Course Code: AUMath-203

Unit-1

- Classification of second order partial differential equations.
- Canonical forms, canonical form for hyperbolic equation, canonical form for parabolic• equation.
- Canonical form for elliptic equation

Unit-11

- Adjoint operators
- Occurrence of the Laplace and Poisson equations, derivation of Laplace equation
 - derivation of Poisson equation
- Boundary value problems, properties of Harmonic functions, separation of variables

Unit-III

- Occurrence of the diffusion equation, boundary conditions, elementary solutions of the
 - diffusion equation.
- Dirac delta function, separation of variables method, solution of diffusion equation in cylindrical coordinates.
- Solution of diffusion equation in spherical coordinates, maximum minimum principle and its consequences.

Unit-IV

- Occurrence of the wave equation, derivation of one dimensional wave equation, solution
 - of one dimensional wave equation by canonical reduction.
- The initial value problem, D'Alembert's solution.
- Boundary and initial value problem for two dimensional wave equation, method of Eig
 - function.
- Periodic solution of one dimensional wave equation in cylindrical coordinates.
- Periodic solution of one dimensional wave equation in spherical polar coordinates,

Suggested Books

1. K. Sankara Rao, Introduction to Partial Differential Equations, Prentice hall of India Private Limited, New Delhi, 1997.

Paper IV: Classical Mechanics

Course Code: AUMath-204

Unit-I

- Generalized coordinates, constraints, work and potential energy, generalized forces.
- The principle of virtual work.
- Introduction to Lagrange's equations, Lagrange's equations for a particle in a plane

Unit-II

- The classification of dynamical system, Lagrange's equations for any simple dynamical system.
- Lagrange's equations for non-holonomic systems with moving constraints.
- Lagrange's equations for impulsive motion.

Unit-III

- Hamilton's principle, stationary values of a function, constrained stationary values.
- Stationary value of a definite integral, the Brachistochrone problem,
- Hamilton's equations, derivation of Hamilton's equations.

Unit-IV

- The form of hamiltonian function, modified hamilton's principle, principle of least action, the Hamilton-Jacobi equation.
- Lagrange's and poisson brackets, calculus of variations.
- Invariance of Lagrange and poisson brackets under canonical transformations.

- 1. Classical Dynamics, Donald. T. Green Wood, Prentice Hall of India, 1979.
- 2. Classical Mechanics, K. Sankara Rao, Prentice Hall of India, 2005.

Paper V: Solid Mechanics

Course Code: AUMath-205

Unit-1

- Analysis of strain, affine transformation, infinitesimal affine deformation
- Geometrical interpretation of the components of the strain, strain quadric of Cauti principle strains.
- Invariants, general infinitesimal deformation, equation of compatibility, finite deformation

Unit-II

- Analysis of stress, stress tensor, equations of equilibrium, stress quadric of Cauchy principle stress and invariance.
- Maximum normal and shear stresses, Mohr's circle diagram.
- Equations of elasticity, generalized Hooke's Law.
- Stress strain relations for a medium having one plane elastic symmetry.

Unit-III

- Strain energy functions and its connection with Hooke's Law.
- Unique solutions of boundary value problem.
- Derivation of Navier equations and Beltrami-Michal compatibility equations.

Unit-IV

- Statement of problem, extension of beams by longitudinal forces, beams stretched by own weight.
- Bending of beams by terminal couples.
- Torsion of a circular shaft.
- Plane stress, plane strain.

Suggested Books

• I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw-Hill Publishing company Ltd.1977.

M.Sc. Mathematics 3 Semester

Paper I: Complex Analysis-I

Course Code: AUMath-301

Unit-1

- The algebra and geometric representation of complex numbers.
- Limit and continuity
- Analytic functions, polynomial and rational functions, the exponential and the trigonometric functions.
- The periodicity, the Logarithm.
- Arcs and closed curves

Unit-II

- Analytic functions in region.
- Conformal mappings, length and area.
- The linear group, the cross ratio, symmetry, oriented circles.
- The use of level curves, a survey of elementary mappings, elementary Riemann
- Line integrals, rectifiable arcs, line integral as function of arcs, cauchy's theorem rectangle, Cauchy's theorem in a disk.

Unit-III

- The index of a point with respect to a closed curve.
- The integral formula.
- Sequences, series, uniform convergence, power series and Abel's limit theorem
- Weierstrass's theorem, the Taylor's series and the Laurent series.
- Removable singularities.
- Taylor's theorem, zeros and poles.
- The local mapping and the maximum principle

Unit-IV

- Chains and cycles, simple connectivity, Homology, the general statement theorem.
- Proof of cauchy's theorem.
- Locally exact differentials and multiply connected regions.
- The residue theorem, the argument principle and evaluation of definite in

Suggested Books

1. Lars v. Ahlfors, Complex Analysis, McGraw Hill Int. Ed. (1979)

Paper II: Topology

Course Code: AUMath-302 Unit-1

- Partial ordered sets and lattices.
- Open sets, closed sets, convergence, completeness.
- Baire's Category theorem, continuity.
- The definition and some examples, elementary concepts, open bases and open subbase Weak topologies, the function algebras C(X,R) and C(X,C).

Unit-II

- Compact spaces, product of spaces.
- Tychonoff's theorem and locally compact spaces.
- Compactness for metric spaces, Ascoli's theorem.

Unit-III

- T1-spaces and Hausdorff spaces.
- Completely regular spaces and normal spaces.
- The Urysohn imbedding theorem.

Unit-IV

- Connected spaces, the components of a space.
- Totally disconnected spaces, locally connected spaces.
- The Weierstrass approximation theorem.

Suggested Books

1. G.F. Simmons, Introduction to Topology and Modern Analysis, International Student Edition, McGraw Hill Book Company, 1963.

Paper III: Analytic Number Theory

Course Code: AUMath-303

Unit-1

- Division Algorithm, The Greatest Common Divisor, The Euclidean Algorithm, and
- The fundamental theorem of Arithmetic.
- The Sieve of Eratosthenes and the Goldbach conjecture.

Unit-II

- Basic properties of Congruence.
- Special divisibility tests and Linear Congruences.
- Fermat's Factorization Method.
- The Little Theorem and Wilson's Theorem.
- The functions and the Mobius Inversion Formula

Unit-III

- The greatest Integer Function and An Application to the Calendar.
- Euler's Phi- Function, Euler's Theorem and some properties of the Phi- Function
- An application to Cryptography,
- The order of an Integer modulo, Primitive roots for Primes.

Unit-IV

- Complex numbers having Primitive roots and the theory of Indices.
- Euler's Criterion.
- The Legendre symbol and its properties.
- Quadratic Reciprocity and Quadratic Congruences with Composite Moduli.

Suggested Books

1. David M. Burton, Elementary Number Theory, (Fifth Edition) International Edition, McGraw Hill.

Paper IV: Operation Research-II

Course Code: AUMath-304 Unit-I

- Queuing systems. Queuing problem. Transient and steady states,
- Theory of Probability, distributions in Queuing systems.
- Poisson process (pure birth process), Properties of passions arrivals
- Exponential process, Markovian property.
- Pure death process, Service time distribution. Erlang service time distribution
- Solution of Queuing Models

Unit-II

- Formulation of LPP. General nonlinear LPP. Constrained optimization with optimization equality constraint, Necessary and sufficient condition for a general NLPP one con SPP (with constraints) with m(<n) constraints.
 - Constrained optimization with inequality constraints (Kuhn Tucker conditions),
 - Saddle point problem, saddle point and NLP.
 - Graphical solution of NLPP. verification of Kuhn Tucker conditions, Kuhn Tucker conditions with non negative constraints.

Unit-III

- Sequencing problems
- Network analysis-PERT and CPM
- Inventory control.

Unit-IV

- Quadratic programming: Wolfe's modified simplex method, Beale's method.
- Separable programming, piecewise linear approximation, separable programming algorithm
- Definition, types of simulation, event type simulation, generation of random number monte-carlo simulation.

- 1. S.D. Sharma, Operations Research, Kedar Nath Ram Nath & Co. 14th Edition
- 2. Kanti Swarup, P.K. Gupta and Manmohan, Operations Research, Sultan Chand & 12th Edition, 2004.

Paper V Mathematical Statistics

Course Code: AUMath 305

- The probability set Function, random variables.
- The probability density Function, the distribution function, Certain probability Mode Mathematical Expectation, Some special Mathematical expectations.

Unit-II

- Chebyshev's Inequality, conditional probability, Marginal and conditional distributions the correlation coefficient. Stochastic Independence,
- The Binomial, trinomial, and Multinomial Distribution, the Poisson distribution
- The Gamma and Chi-square Distributions, the normal distribution, and the hivariste normal distribution.

Unit-II

- Sampling theory, Transformations of variables of the Discrete type, Transformations of the variables of the continuous type.
- The t and F distributions. Extensions of the change of variable Technique, Distributions Of order statistics, the moment generating function Technique.

Unit-IV

- The distribution of and 2/02, Expectations of Functions of Random variables.
 Limiting Distributions
- Stochastic Convergence, Limiting Moment Generating Functions. The Central limit Theorem, some theorems on limiting Distributions,

Suggested Books

1. Robert V. Hogg and Allen T. Craig, Introduction to Mathematical Statistics, Forth Edition, Macmillan Publishing Co., Inc., New York, 1989.

M.Sc. Mathematics 4 Semester

Paper 1: Complex Analysis-II

Course Code: Math 401

Unit-1

- Definition and basic properties of harmonic function, The mean value property
- Hadamard's three circle theorem. Poisson formula Schwarz's theorem, the reflection Principle.
- A closer look at Harmonic functions, Functions with the mean value property.
- Harnack's principle, The Dirichlet's problem: Subharmonic functions, Solution of Dirichlet's problem.

Unit-II

- Partial fractions, infinite products, onical products, the Gamma functions.
- Stirling formula, Entire functions: Jensen's formula, Hadamard's theorem.
- The Riemann zeta functions. The product development. Extension of () to the whole plane. The functional equation. The zeros of the zeta function.

Unit-III

- Simply periodic functions. Representation by exponentials, The Fourier development, Functions of finite order.
- Doubly periodic functions, the period module, unimodular transformations. The canonical basis, General properties of elliptical functions.

Unit-IV

- Analytic continuations, The Weierstrass theory, germs and sheaves, Sections and Riemann surfaces, Analytic continuations along ares, Homotopic curves, The Monodromy theorem, Branch points.
- Algebraic functions, the resultant of two polynomials, Definition and properties of algebraic functions, behaviour at the critical points.
- Picard's theorem.

Suggested BookS

1.L Lars V. Ahlfors, Complex Analysis, Int. Ed. McGraw-Hill Book Co. (Third Edition). (1979),

Paper II: Function Analysis

Course Code: Math-402

Unit-1

- The definition and some examples, continuous linear transformations.
- The Hahn-Banach Theorem, the Open Mapping Theorem, the Closed Graph Theorem, the Uniform Boundedness Theorem, the natural embedding of N in N, reflexivity.
- Dual space, <u>L.PL</u> duality

Unit-II

- Baire's theorem, open mapping, closed graph
- Uniform boundedness, principal & application to fourier series
- Multi-linear algebra- tensor, symmetric and exterior products.

Unit-III

- The definition and some simple properties, orthogonal complements, orthonormal sets,
- The conjugate space H, the adjoint of an operator, self-adjoint normal and unitary operators, projections.

Unit-IV

- Spectral Theory in Finite Dimensional Normed Spaces, Basic Concepts.
- Spectral Properties of Bounded Linear Operators.
- Further Properties of Resolvent and Spectrum.
- Use of Complex Analysis in Spectral Theory, Banach Algebras, Further Properties Banach Algebras.

- 1. G.F. Simmons, Introduction to Topology and Modern Analysis, International Student Edition, McGraw Hill Book Company, Inc. 1963.
- 2. E. Kreyszig. Introductory Functional Analysis with Applications, John, Wiley and Sons Wiley Classics Library Edition Published, 1989.

Paper III: Advanced Discrete Mathematics

Course Code: AUMath-403

Unit-1

- Logic, Propositional Equivalences, Predicates and Quantifiers.
- Partial Ordered Sets. Lattices and Algebraic Systems. Principle of Duality, Basic Properties of Algebraic Systems defined by Lattices, Distributive and Complemented Lattices.
 - Boolean Lattices and Boolean Algebras, Uniqueness of Finite Boolean Algebras Boolean Functions and Boolean Expressions.

Unit-11

- Pigeonhole principle: Simple form, Pigeonhole principle: Strong form. A theorem of Ramsey.
- Two basic counting principles, Permutations of sets, Combinations of Sets.
- Permutations of multi sets, Combinations of multisets.

Unit-II

- Generating permutations, Inversions in permutations, Generating combinations, Partial orders and equivalence relations.
- Pascal's formula, the binomial theorem, Identities, Unimodality of binomial coefficient
- The multinomial theorem, Newton's binomial theorem.
- The inclusion-exclusion principle, Combinations with repetition, Derangements,
- Permutations with forbidden positions.
- Some number sequences, linear homogeneous recurrence relations.
- Non-homogeneous recurrence relations, Generating functions. Recurrences and generating functions, Exponential generating functions.

Unit-IV

- Basic properties, Eulerian trails, Hamiltonian chains and cycles, Bipartite multigraphs, Trees, The Shannon switching game.
- Digraphs and Networks.
- Chromatic number, Plane and planar graphs, A 5-color theorem.

- 1. C.L. Liu, Elements of Discrete Mathematics'. Tata McGraw-Hill, Second Edition.
- 2. Richard A. Brualdi. Introductory Combinatorics, third Edition.

Paper IV: Differential Geometry

Course Code: AUth-404

Unit-1

- Tangent. Principal normal. Curvature,
- Binormal. Torsion, Serret- Frenet formulae
- Locus of center of curvature, Spherical curvature, Locus of center of spherical curvature.
- . Curve determined by its intrinsic equations.

Unit-11

- Helices, Involutes & Evolutes.
- Surface, Tangent plane, Normal, Curvilinear coordinates.
- First order magnitude, Directions on a surface. The normal
- Second order magnitudes, Derivatives of n. Curvature of normal section.

Unit-III

- Menger's theorem, Principal directions and curvature, first and second curvature,
- Euler's theorem.
- Surface of revolution.

Unit-1V

- Gauss's formulae, Gauss characteristic equation, Mainardi Codazzi relations.
- Derivatives of angle w.
- Geodesic property, Equations of geodesics, Surface of revolution, Torsion of Geodesi
- Bonnet's theorem, vector curvature, Geodesic curvature.

Suggested Book

1. Differential Geometry of Three Dimension, C.E. Weatherburn, Khosla Publishing House, 2003.

Paper V: Magneto Fluid Dynamics

Course Code: AUMath-405

Unit-1

- Maxwell's electromagnetic field equations, Magnetic induction equation induction and magnetic Reynold's number.
- Alfven's Theorem and its consequences.
- Magnetic energy equation.
- Mechanical questions and effects.

Unit-11

- Magneto hydrostatic, Force Free magnetic fluids (Basic equations, boundary con &magnetic energy, general solution when a is constant).
- Pressure balanced magneto hydrostatic configurations.
- Toroidal magnetic field.

Unit-III

- Steady laminar motion.
- General solution of a vector wave equation.
- Alfven waves, Magneto hydrodynamic waves in compressible fluid.
- Reflection and refraction of Alfven waves. Dissipative effects.

Unit-IV

- Introduction, Linear Pinch.
- Method of small Oscillations Energy principle. Virial theorem. Marginal stability Benard problem with a magnetic field.
- Introduction, spectral analysis. Homogeneity and Isotropy. Kolmogorov principle.
- Hydro magnetic turbulence. Inhibition of turbulence by a magnetic field.

Suggested Books

1. An Introduction to Magneto Fluid Dynamics by V.C.A. Ferraro & C. Plumpton. Clarendon Press, Oxford 2nd Edition, 1966.